$$A=\frac{u\sqrt{2}}{\sqrt{(a^2-r^2u^2)}}. \quad \therefore \sigma=\left[\frac{a}{u}\tan^{-1}\left(\frac{\sqrt{(a^2-r^2u^2)}-\sqrt{(a^2+r^2u^2)}}{ru\sqrt{2}}\right)\right]_{r=0}^{r=a}$$

$$\therefore \sigma=\frac{a}{u}\tan^{-1}\left(\frac{\sqrt{(1-u^2)}-\sqrt{(1+u^2)}}{u\sqrt{2}}\right)=120\tan^{-1}\left(\frac{\sqrt{11}-\sqrt{61}}{5\sqrt{2}}\right)$$

$$=314\tfrac{1}{3}\text{ feet, nearly.}$$

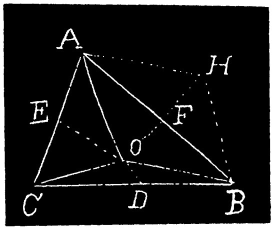
Prof. G. W. Greenwood solves the problem, interpreting it according to Prof. Zerr's assumption, but he gets a slightly different result. The Proposer furnished a partial solution for *The Educational Times*, London, obtaining the same differential equation as that obtained by Prof. Zerr, but by a different method.

MECHANICS.

150. Proposed by G. B. M. ZERR. A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

O is a point in a plane of a triangle, ABC, and D, E, F are the mid-points of the sides. Show, geometrically, that the system of forces OA, OB, OC is equivalent to the system OD, OE, OF.

Solution by G. W. GREENWOOD, B. A., Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.



Complete the parallelogram $AOBH$, of which OE is one-half the diagonal.

Since the forces OH, OB, are equivalent to OH, $\frac{1}{2}OA$ and $\frac{1}{2}OB$ will be equivalent to OF.

Similarly, $\frac{1}{2}OB$ and $\frac{1}{2}OC$ will be equivalent to OD, and $\frac{1}{2}OC$ and $\frac{1}{2}OA$ will be equivalent to OE.

Hence the system OA, OB, OC is equivalent to the system OD, OE, OF.

151. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

An elastic ball is projected along a horizontal tube, striking first the bottom, then the top, then the bottom and so on. Find the number of times the ball will strike the top.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let v=velocity just before first impact, r=diameter of tube, $v_1=\sqrt{(2gr)}$, e=coefficient of restitution. Then velocity of leaving bottom$=ev$, velocity of arrival at top$=ev-v_1$, velocity of arrival at bottom$=e^2v-ev_1+v_1$, velocity of second arrival at top$=e^3v-e^2v_1+ev_1-v_1$, velocity of third arrival at top$=e^5v-e^4v_1+e^3v_1-e^2v_1+ev_1-v_1$.

$\therefore$ Velocity of nth arrival at top$=e^{2n-1}v-v_1(e^{2n-2}-e^{2n-3}+e^{2n-4}-e^{2n-5}+\ldots+e^4-e^3+e^2-e+1)$, and this velocity$=v_1$.

$\therefore e^{2n-1}v-v_1(e^{2n-2}+e^{2n-4}+\ldots+e^4+e^2)+v_1(e^{2n-3}+e^{2n-5}+\ldots+e^3+e)=2v_1$.

$\therefore\ e^{2n-1}v-e^2v_1\left(\frac{e^{2n-2}-1}{e^2-1}\right)+ev_1\left(\frac{e^{2n-2}-1}{e^2-1}\right)=2v_1.$

$\therefore\ v(e^2-1)e^{2n-1}-ev_1e^{2n-1}+v_1e^{2n-1}=v_1(e^2+e-2).$

$\therefore\ e^{2n-1}=\frac{v_1(e+2)}{v(e+1)-v_1}=A$, suppose.

$\therefore\ 2n-1=\log A/\log e$. $\therefore\ n=\frac{1}{2}\log(Ae)/\log e=$the number required.

DIOPHANTINE ANALYSIS.

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107. **Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.**

Required the least three positive integral numbers such that the sum of all three of them, and the sum of every two of them shall be a square number.

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let x, y, and z =the numbers.

Then $x+y=\square=h^2$, $x+z=\square=k^2$, $y+z=\square=l^2$.

$\therefore\ x+y+z=\frac{1}{2}(h^2+k^2+l^2)=\square=s^2$....(1).

$\therefore\ x=s^2-l^2$, $y=s^2-k^2$, and $z=s^2-h^2$.

Put $l=s-m$, $k=s-n$, and $h=s-r$. Substituting these values in (1), we obtain $3s^2-2s(m+n+r)+m^2+n^2+r^2=2s^2$.

Solving for s, we find $s=m+n+r\pm\sqrt{[2(mn+mr+nr)]}$.

Take $mn+mr+nr=mn+r(m+n)=2b^2$.

$\therefore\ s=m+n+x\pm2b$,

$x=2ms-m^2=m(2s-m)$....(2),

$y=2ns-n^2=n(2s-n)$....(3),

$z=2rs-r^2=r(2s-r)$....(4).

Put $n=m+a$. Then $mn+r(m+n)=m(m+a)+r(2m+a)=2b^2$.

$\therefore\ r=\frac{2b^2-m(m+a)}{2m+a}$, and $s=\frac{(m+a)(3m+a)-am+2b[b\pm(2m+a)]}{2m+a}$.

Substituting in (2), (3), and (4), and multiplying by $(2m+a)^2$, we obtain the following general values:

$$x=m(2m+a)\{2(m+a)(2m+a)-am+4b[b\pm(2m+a)]\},$$
$$y=(m+a)(2m+a)\{(2m+a)^2-am+4b[b\pm(2m+a)]\},$$
$$z=[2b^2-m(m+a)]\{2[b\pm(2m+a)]^2-m(m+a)\},$$
$$x+y=\{(2m+a)[2b\pm(2m+a)]\}^2,$$
$$x+z=\{m^2+2b[b\pm(2m+a)]\}^2,$$
$$y+z=\{(m+a)^2+2b[b\pm(2m+a)]\}^2,$$
$$x+y+z=\{(m+a)(3m+a)-am+2b[b\pm(2m+a)]\}^2.$$

For *positive* values, we have the general condition, $2b^2>m(m+a)$; also, when $b-(2m+a)$ is used, the condition, $b>2m+a$.

When x, y, and z have a common divisor, lowest values are obtained by dividing by the highest common *square* factor.

Multiple values may be obtained by multiplying any set of values of x, y, and z by a *square* number.